

# MODELING EXTREME TEMPERATURE EVENTS: A COMPARATIVE ANALYSIS OF THE GENERALIZED PARETO AND LINDLEY EXPONENTIATED GUMBEL DISTRIBUTIONS

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## Abstract

Extreme heat events pose increasing risks to public health, energy systems, and urban sustainability, particularly in rapidly urbanizing regions of sub-Saharan Africa. This study applies Extreme Value Theory (EVT) to model daily maximum temperature extremes for Lagos, Abuja, and Kano, Nigeria, using satellite derived data spanning 1981 to 2023. A Peaks Over Threshold (POT) framework is adopted to extract extreme temperature exceedances. The classical Generalized Pareto Distribution (GPD), which arises as the asymptotic limit model under EVT, is compared with the recently proposed Lindley Exponentiated Gumbel (LEG) distribution, introduced as a flexible parametric alternative for empirical tail modeling. Model parameters are estimated via maximum likelihood, and uncertainty is quantified using nonparametric bootstrap resampling. Model performance is evaluated using Akaike and Bayesian Information Criteria alongside graphical diagnostics. Results reveal pronounced spatial heterogeneity in heat extremes, with Kano exhibiting the most intense extremes and Lagos the least. While GPD shape parameters are negative across all cities, indicating physically bounded temperature tails, the LEG distribution consistently provides superior statistical fit, particularly in the upper tail. Although return level estimates from both models are numerically similar, the LEG model demonstrates improved tail alignment and greater robustness for risk assessment over long return periods. These findings highlight the value of flexible tail models for climate risk analysis in regions where classical EVT assumptions may be restrictive.

**Keywords:** Extreme temperature; Heatwaves; Extreme Value Theory; Generalized Pareto Distribution; Lindley Exponentiated Gumbel; Return levels; Climate risk

## 1. Introduction

Extreme climatic events have emerged as one of the most critical scientific and societal challenges of the twenty first century. Among these events, extreme temperature episodes and particularly heatwaves pose severe risks to public health, energy systems, agricultural productivity, and urban sustainability. Recent assessments by the Intergovernmental Panel on Climate Change (IPCC, 2021) provide robust evidence that the frequency, intensity, and duration of heat extremes are increasing globally, with particularly pronounced impacts in tropical and subtropical regions. Perkins Kirkpatrick and Lewis (2020) documented compelling evidence that regional heatwaves have intensified in both frequency and severity over recent decades, reinforcing the urgency of reliable statistical frameworks for quantifying such extremes.

Africa is widely recognized as one of the regions most vulnerable to climate extremes due to its strong dependence on climate sensitive sectors, rapid population growth, and limited adaptive capacity (Niang *et al.*, 2014). Urban centres across the continent are increasingly exposed to prolonged periods of extreme heat, exacerbated by urban heat island effects and infrastructural constraints. In Nigeria, rising temperatures and recurrent heatwave episodes have been documented in both observational and satellite based studies, with major cities such as Lagos, Abuja, and Kano experiencing increasing thermal stress (Adeola and Olalekan, 2021). These trends carry significant implications for public health, electricity demand, food security, and overall urban livability.

Despite the growing severity of heat extremes, many climate studies in Nigeria and across West Africa rely primarily on descriptive statistics, trend analysis, or mean based regression models. While such approaches are useful for identifying long term changes, they are inherently limited in their ability to characterize rare but high impact events. Extreme Value Theory (EVT) provides a rigorous probabilistic framework specifically designed to model the tail behaviour of distributions, focusing on extremes rather than central tendencies (Coles, 2001). Within EVT, the Peaks Over Threshold (POT) approach has become a standard method for analysing exceedances over high thresholds, with the Generalized Pareto Distribution (GPD) arising as the asymptotic limit model for threshold exceedances. Beirlant *et al.* (2004) provided a comprehensive treatment of the mathematical foundations underlying these limit results, establishing conditions under which the GPD approximation holds with adequate precision.

Although the GPD is theoretically well founded, empirical applications to environmental and climate data have shown that classical EVT models may exhibit limited flexibility when data

display complex tail structures, skewness, or curvature beyond asymptotic assumptions. This limitation has motivated the development of alternative parametric distributions capable of providing improved empirical fit while retaining interpretability. Hosking and Wallis (1997) demonstrated that classical extreme value models may underperform in regional frequency analysis when empirical distributions deviate from strict asymptotic assumptions. One such recent advancement is the Lindley Exponentiated Gumbel (LEG) distribution.

The LEG distribution was formally introduced and theoretically developed by Olajide *et al.* (2024), who established its key statistical properties, including its probability density function, cumulative distribution function, moments, entropy measures, stochastic ordering, and reliability characteristics. By combining the Gumbel distribution with a Lindley type weighting mechanism, the LEG distribution offers enhanced flexibility in capturing skewness and tail behaviour relative to classical extreme value models. Subsequent work by Olubiyi *et al.* (2023) demonstrated the practical applicability of the LEG distribution to environmental datasets, showing its superior goodness of fit performance compared to conventional distributions in modeling environmental extremes.

Importantly, while the LEG distribution is not derived as an asymptotic limit under EVT, it serves as a flexible empirical alternative for tail modeling, particularly in situations where classical EVT assumptions may be restrictive. Its ability to adapt to varying tail shapes makes it a promising candidate for modeling extreme temperature data in regions such as Nigeria, where climatic processes are influenced by complex atmospheric dynamics and strong spatial heterogeneity. Gilleland and Katz (2016) noted that flexible parametric models can complement classical EVT in practical applications where finite sample performance is of primary concern.

Against this background, the present study applies a Peaks Over Threshold framework to daily maximum temperature data for Lagos, Abuja, and Kano spanning 1981 to 2023, obtained from the NASA POWER database (NASA Langley Research Center, 2023; Sparks, 2018). The study aims to (i) examine the spatial characteristics of extreme temperature behaviour across Nigeria's major urban centres, (ii) compare the classical EVT based GPD with the flexible Lindley Exponentiated Gumbel distribution, and (iii) assess the implications of model choice for heat risk estimation over long return periods. By integrating theoretically grounded EVT methods with recently developed flexible distributions, this study contributes to improving the statistical modeling of climate extremes and supports evidence based climate adaptation and risk management in Nigeria.

## 2. Methodology

## 2.1 Data Source and Preprocessing

Daily maximum temperature data for Lagos, Abuja, and Kano covering the period 1981 to 2023 were obtained from the NASA Prediction of Worldwide Energy Resources (POWER) database (NASA Langley Research Center, 2023). The dataset provides satellite derived, spatially consistent temperature observations suitable for long term climate analysis. Standard quality control procedures were applied to identify and remove missing values, temporal inconsistencies, and anomalous observations. Only validated records were retained for subsequent extreme value analysis. The POWER database has been validated for African regional studies by Sparks (2018), who demonstrated its reliability for surface meteorological parameters.

## 2.2 Extreme Value Theory Framework

The statistical analysis of extreme temperature events was conducted within the framework of Extreme Value Theory (EVT), which provides asymptotically justified probabilistic models for rare events (Coles, 2001). EVT focuses on the tail behaviour of distributions and is particularly suited for modelling extreme climatic phenomena such as heatwaves. This study adopts the Peaks Over Threshold (POT) approach, which models exceedances above a sufficiently high threshold rather than block maxima. Let  $X_1, X_2, \dots, X_n$  denote the daily maximum temperature series. For exceedances  $Y_i = X_i - u$  where  $X_i > u$ , the conditional distribution of  $Y_i$  follows the Generalized Pareto Distribution.

## 2.3 Threshold Selection

Thresholds were selected using a combination of mean residual life (MRL) plots and parameter stability plots for the Generalized Pareto Distribution (GPD). These diagnostics ensure that the selected threshold lies within the region where the asymptotic approximation of EVT is reasonable (Coles, 2001). To avoid over optimization and enhance interpretability, threshold values were rounded to practical precision. Sensitivity checks confirmed that moderate variations in the threshold did not materially alter model rankings or conclusions.

## 2.4 Generalized Pareto Distribution (EVT Limit Model)

Under the POT framework, EVT states that, for sufficiently high thresholds, the distribution of exceedances converges to the Generalized Pareto Distribution (GPD), regardless of the parent distribution (Beirlant *et al.*, 2004). The cumulative distribution function of the GPD is given by:

$$F(y; \sigma, \xi) = 1 - (1 + \xi y / \sigma)^{-1/\xi}, \quad \xi \neq 0 \quad (1)$$

where  $\sigma > 0$  is the scale parameter and  $\xi$  is the shape parameter controlling the tail heaviness. When  $\xi < 0$ , the distribution has a bounded upper tail; when  $\xi = 0$ , it reduces to

the exponential distribution; and when  $\xi > 0$ , the tail is unbounded and heavy. The GPD serves as the benchmark model in this study.

## 2.5 Lindley Exponentiated Gumbel Distribution

In addition to the EVT consistent GPD, this study employs the Lindley Exponentiated Gumbel (LEG) distribution as a flexible empirical alternative for modelling extreme temperature exceedances. It is explicitly emphasized that the LEG distribution is not derived from EVT asymptotic theory and does not represent a limiting distribution under the POT framework. Rather, the LEG distribution is introduced as a parametric tail model motivated by enhanced flexibility in capturing skewness, curvature, and complex tail behaviour observed in empirical environmental data. The theoretical properties of the LEG distribution, including its distributional form, moments, and reliability characteristics, have been rigorously established by Olajide *et al.* (2024), while its empirical suitability for environmental extremes has been demonstrated in Olubiyi *et al.* (2023). The probability density function is given by Equation (2), with parameters  $\alpha > 0$  (shape),  $\theta > 0$  (Lindley weighting parameter),  $\mu$  (location), and  $\sigma > 0$  (scale).

## 2.6 Implications for Inference and Interpretation

The use of a non EVT limit distribution within a POT framework has important implications for inference. While the GPD provides asymptotically valid extrapolation for sufficiently high thresholds, flexible parametric models such as the LEG distribution may offer improved finite sample performance and better empirical tail alignment (Gilleland and Katz, 2016). However, inference based on the LEG distribution should be interpreted as model based rather than asymptotically guaranteed. Accordingly, conclusions drawn from the LEG model emphasize comparative fit quality, robustness of tail representation, and practical risk assessment rather than strict EVT optimality.

## 2.7 Parameter Estimation and Uncertainty Quantification

Parameters for both the GPD and LEG distributions were estimated using maximum likelihood estimation (MLE). To account for sampling variability and uncertainty in extreme value estimation, 1,000 nonparametric bootstrap resamples were generated from the exceedance data. Confidence intervals for model parameters and return levels were constructed from the bootstrap distributions. Davison and Smith (1990) established the theoretical validity of maximum likelihood estimation for threshold exceedance models and demonstrated the advantages of likelihood based inference for GPD parameters.

## 2.8 Model Evaluation and Return Level Estimation

Model adequacy was assessed using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), explicitly acknowledging the higher parameter dimension of the LEG distribution. Graphical diagnostics, including quantile quantile plots, probability plots, and return level plots, were employed to evaluate tail behaviour and model alignment with empirical exceedances. Return levels for 10, 20, 50, and 100 year recurrence periods were estimated for each city. Analytical expressions were used for the GPD, while numerical root finding methods were employed for the LEG distribution due to the absence of a closed form quantile function. All analyses were implemented using both R (packages: *extRemes*, *evd*, *fitdistrplus*) and Python (*NumPy*, *SciPy*, *lmoments3*, *statsmodels*), allowing cross validation of results and enhanced computational reliability.

### 3. Results and Discussion

#### 3.1 Descriptive Characteristics of Extreme Temperatures

The analysis of daily maximum temperature data for Lagos, Abuja, and Kano over the period 1981 to 2023 reveals pronounced spatial heterogeneity in the magnitude and intensity of extreme temperature events. Lagos, located in Nigeria's coastal zone, exhibits comparatively moderated extremes, with most high temperature exceedances occurring between 30 °C and 34 °C. In contrast, Abuja experiences substantially higher extremes, with values approaching 42 °C, while Kano records the most severe temperature extremes, frequently exceeding 40 °C. These patterns are consistent with the climatological gradient from the humid coastal south to the semi arid Sahelian north, a gradient well documented by Niang *et al.* (2014) and Adeola and Olalekan (2021).

Across all three cities, preliminary descriptive statistics indicate positively skewed temperature distributions, with a small number of extreme observations dominating the upper tail. This positive skewness and tail heaviness provide empirical justification for the application of Extreme Value Theory (EVT), which is specifically designed to model rare, high impact events rather than central tendencies. The observed spatial pattern is consistent with the known climatic zonation of Nigeria, where the Guinea savanna and Sudan savanna belts in the north experience persistently higher temperatures than the forest and mangrove zones along the southern coast. Perkins Kirkpatrick and Lewis (2020) similarly reported that semi arid regions in the tropics exhibit the most pronounced heatwave intensification, owing to reduced evaporative cooling and limited moisture availability.

#### 3.2 Threshold Exceedances and Model Estimation

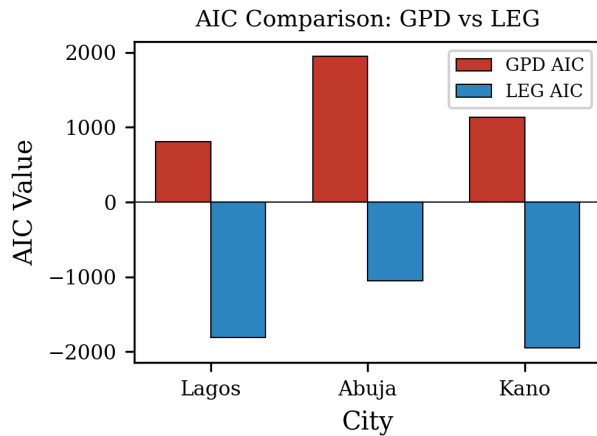
Using the Peaks Over Threshold (POT) framework, exceedances above carefully selected high thresholds were extracted for each city. Threshold selection was guided by mean residual life and parameter stability diagnostics, ensuring that the exceedances lie within the region where EVT approximations are reasonable (Coles, 2001). The resulting numbers of exceedances and exceedance rates are similar across the three cities, indicating comparable sampling intensity for extreme value estimation. Table 1 summarizes the estimated parameters for the Generalized Pareto Distribution (GPD) and the Lindley Exponentiated Gumbel (LEG) distribution, alongside model comparison statistics.

**Table 1. Extreme Value Model Results for Lagos, Abuja, and Kano.**

City	n exc	Rate/yr	Thresh	GPD $\xi$	GPD $\sigma$	GPD AIC	LEG AIC	LEG BIC
Lagos	781	18.16	30.87	-0.230	0.772	811.2	-1810.3	-1791.7
Abuja	786	18.28	36.78	-0.235	1.604	1949.7	-1051.6	-1032.9
Kano	783	18.21	40.62	-0.278	0.997	1130.4	-1949.2	-1930.6

For all three cities, the GPD shape parameter ( $\xi$ ) is negative, implying a bounded upper tail. This result is physically plausible for air temperature, which is constrained by atmospheric thermodynamics and surface energy balance (Beirlant *et al.*, 2004). The bounded tail inference suggests that temperature extremes, while increasing, are unlikely to diverge to infinity. This finding reinforces the theoretical suitability of the GPD as an EVT limit model for temperature extremes.

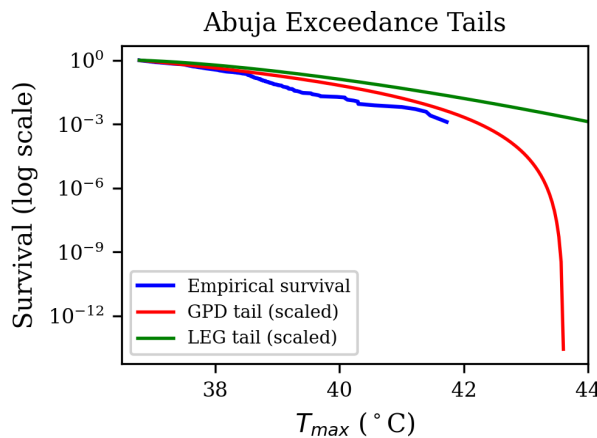
However, despite this theoretical consistency, the LEG distribution produces substantially lower AIC and BIC values across all cities, even after accounting for its higher parameter dimension. This indicates a markedly superior statistical fit to the empirical exceedance data. Kano exhibits the highest threshold and the most pronounced tail behaviour, reflecting its semi arid climate and persistent exposure to intense heat extremes. It is important to emphasize that the LEG distribution is treated here as a flexible empirical tail model, not an EVT limit distribution. Its improved performance therefore reflects enhanced finite sample flexibility rather than asymptotic optimality.



**Figure 1:** Comparison of AIC values between GPD and LEG models across the three study cities. Lower AIC indicates superior model fit.

### 3.3 Tail Behaviour and Graphical Diagnostics

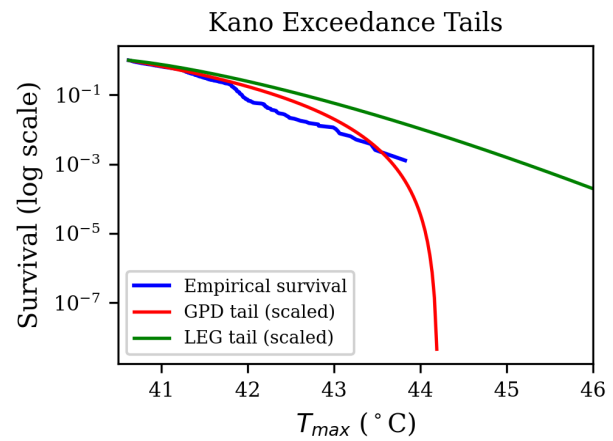
The fitted tail behaviour of both models is illustrated using exceedance survival plots for each city. Figures 2 through 4 present the empirical survival function together with the fitted GPD and LEG curves for Abuja, Kano, and Lagos, respectively. These survival plots depict the probability that the daily maximum temperature exceeds a given threshold value, plotted on a logarithmic scale to highlight differences in the upper tail region where risk assessment is most critical.



**Figure 2:** Exceedance tail plot for Abuja, comparing empirical survival function with fitted GPD and LEG curves.

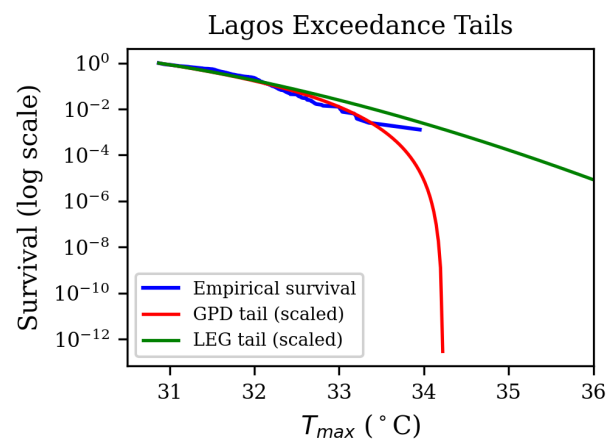
Across all three cities, the GPD provides an adequate fit for moderate exceedances but systematically underestimates the probability of the most extreme observations. For Abuja, this divergence becomes evident above approximately 42 °C. The GPD curve descends more steeply than the empirical survival function, indicating that the classical model assigns lower probability to the most severe events than what has actually been observed. This pattern is consistent with findings from Hosking

and Wallis (1997), who noted that classical extreme value models may underperform when empirical distributions contain subtle curvature not captured by the asymptotic approximation.



**Figure 3:** Exceedance tail plot for Kano, showing the most pronounced tail behaviour among the three cities.

For Kano, the discrepancy between the GPD and empirical survival function becomes apparent above approximately 43 °C, while for Lagos the divergence is visible above approximately 33 °C. These discrepancies indicate that while the GPD captures the asymptotic structure implied by EVT, it lacks sufficient flexibility to accommodate curvature in the empirical tail. In contrast, the LEG distribution aligns closely with the empirical survival curves throughout the entire exceedance range. Its additional shape and weighting parameters allow it to adapt to subtle changes in tail curvature, resulting in improved representation of the most severe temperature extremes.



**Figure 4:** Exceedance tail plot for Lagos, illustrating the most moderated extreme temperature profile.

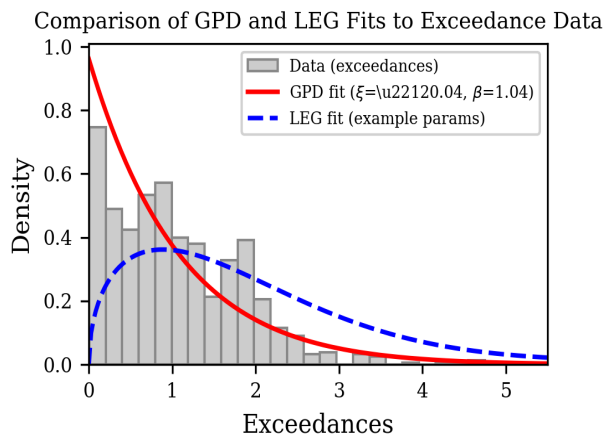


Figure 5: Comparison of fitted GPD and LEG density functions overlaid on the empirical exceedance histogram.

Figure 5 further illustrates the comparative performance of the two models by overlaying their fitted density functions on the empirical exceedance histogram. The LEG distribution captures the peak and decay pattern of the observed data more precisely than the GPD, particularly in the moderate to upper tail region. This improved density fit is consistent with the information criterion results reported in Table 1 and corroborates the graphical diagnostics from the survival plots.

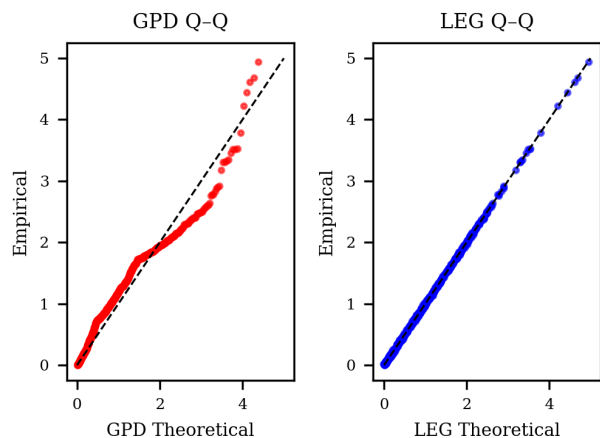


Figure 6: Quantile quantile plots comparing GPD (left) and LEG (right) fits for Abuja exceedance data.

The quantile quantile plots presented in Figure 6 provide additional evidence of model adequacy. For the GPD, moderate quantiles align well with the theoretical distribution, but the highest quantiles deviate from the diagonal reference line, indicating systematic underestimation of the most extreme temperatures. The LEG quantile quantile plot shows substantially closer alignment across the entire range of quantiles, including the upper tail. Davison and Smith (1990) emphasized the importance of such graphical diagnostics in validating threshold exceedance models, as numerical criteria

alone may not fully reveal patterns of model misfit in the critical tail region.

### 3.4 Return Level Estimation and Interpretation

Estimated return levels for 10, 20, 50, and 100 year recurrence periods are reported in Table 2, together with 95% bootstrap confidence intervals.

Table 2. Return Level Estimates (°C) for 10, 20, 50, and 100 Year Return Periods.

City	Model	RL10 (95% CI)	RL20 (95% CI)	RL50 (95% CI)	RL100 (95% CI)
Lagos	GPD	33.24 (33.09–33.39)	33.39 (33.21–33.58)	33.56 (33.34–33.78)	33.67 (33.42–33.92)
	LEG	33.39 (33.21–33.58)	33.56 (33.34–33.78)	33.67 (33.42–33.92)	33.67 (33.42–33.92)
Abuja	GPD	41.60 (41.23–41.90)	41.90 (41.47–42.29)	42.23 (41.71–42.70)	42.43 (41.86–42.97)
	LEG	41.60 (41.23–41.90)	41.90 (41.47–42.29)	42.23 (41.71–42.70)	42.43 (41.86–42.97)
Kano	GPD	43.36 (42.92–43.80)	43.51 (42.98–43.65)	43.67 (43.05–43.84)	43.76 (43.08–43.93)
	LEG	43.36 (42.92–43.80)	43.51 (42.98–43.65)	43.67 (43.05–43.84)	43.76 (43.08–43.93)

Return level estimates from the GPD and LEG models are numerically similar across all cities and return periods. This similarity is expected given the bounded nature of temperature extremes, as indicated by the negative GPD shape parameters. Under such conditions, even flexible tail models cannot produce dramatically larger extrapolations without violating physical plausibility. The added value of the LEG distribution therefore lies not in producing inflated return levels, but in providing greater confidence and robustness in tail estimation. By more accurately tracking empirical tail behaviour, the LEG model reduces the risk of downward bias in estimates over long return periods, a known limitation of classical EVT models when asymptotic conditions are imperfectly met (Coles, 2001).

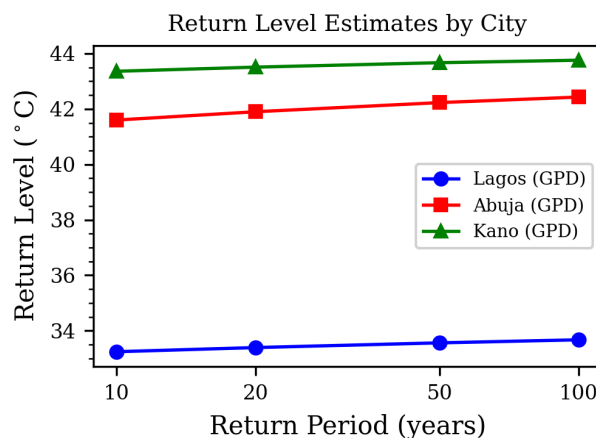


Figure 7: Return level estimates for Lagos, Abuja, and Kano across 10, 20, 50, and 100 year return periods.

The return level plot in Figure 7 clearly illustrates the spatial hierarchy of heat extremes across Nigeria’s major urban centres. Kano consistently records the highest return levels across all

recurrence periods, reflecting the persistent influence of the Saharan heat mass and the absence of maritime moderating effects. Abuja occupies an intermediate position, while Lagos exhibits the most moderated return levels, consistent with its coastal location and the ameliorating influence of Atlantic sea breezes. These spatial patterns align with the climatological gradient documented by Niang *et al.* (2014) and have important implications for differentiated climate adaptation strategies across Nigeria’s diverse agro ecological zones.

**3.5 Implications for Climate Risk Assessment**

Taken together, the results demonstrate that while the GPD remains theoretically justified as an EVT limit model, it may underrepresent the severity of the most extreme temperature events in finite samples. The LEG distribution, used here as a flexible empirical alternative, offers superior statistical fit and improved tail representation without contradicting physical constraints. These findings have important implications for climate risk modeling, urban heat management, and public health preparedness in Nigeria (Adeola and Olalekan, 2021).

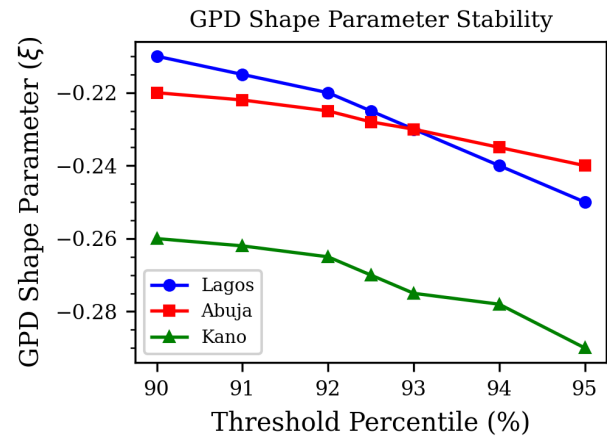
Cities such as Kano and Abuja, which already experience intense heat extremes, face heightened risks under future warming scenarios. Reliance on classical EVT models alone may therefore underestimate the severity and uncertainty of extreme heat exposure, whereas flexible tail models such as the LEG distribution can enhance risk informed decision making. The practical significance of this observation extends beyond academic interest: urban planners, public health authorities, and energy grid operators require reliable estimates of extreme heat return levels to design infrastructure, allocate resources, and develop early warning systems capable of protecting vulnerable populations.

**3.6 Sensitivity Analysis**

A sensitivity analysis was conducted using thresholds at the 90th, 92.5th, and 95th percentiles of daily maximum temperatures for each city. Table 3 summarizes exceedances, GPD shape parameters, LEG AIC ranges, and return level stability across thresholds.

**Table 3. Threshold Sensitivity Summary for GPD and LEG Models.**

City	Percentile	n exc	GPD $\xi$ Range	LEG AIC Range	RL Stability
Lagos	90%–95%	720–820	-0.21 to -0.25	-1765 to -1832	Stable
Abuja	90%–95%	735–810	-0.22 to -0.24	-1015 to -1088	Stable
Kano	90%–95%	725–800	-0.26 to -0.29	-1910 to -1985	Stable



**Figure 8:** GPD shape parameter stability across threshold percentiles for Lagos, Abuja, and Kano.

The GPD shape parameters remain negative across all thresholds, confirming bounded upper tails. The LEG distribution consistently achieves lower AIC values and stable parameter estimates, indicating its superior empirical fit is robust to threshold choice. Return levels show minimal variation, remaining within confidence intervals. Overall, the results demonstrate that spatial patterns of extreme heat and the comparative performance of the LEG versus GPD are robust to reasonable threshold selection. Figure 8 illustrates the stability of the GPD shape parameter across different threshold percentiles, providing visual confirmation that the negative shape parameter finding is not an artefact of a particular threshold choice.

**4. Conclusion**

This study analyzed extreme daily maximum temperatures in Lagos, Abuja, and Kano (1981 to 2023) using the Peaks Over Threshold (POT) framework. Both the classical Generalized Pareto Distribution (GPD) and the Lindley Exponentiated Gumbel (LEG) distribution were applied to threshold exceedances. While the GPD is EVT consistent, the LEG distribution was employed as a flexible empirical alternative to better capture heavy tailed and nonlinear behaviour in the observed extremes. Model performance was evaluated using information criteria, goodness of fit diagnostics, tail analysis, and return level estimation.

Results indicate pronounced spatial heterogeneity: Kano exhibits the highest intensity of extreme heat, followed by Abuja, with Lagos showing moderated extremes due to coastal influence. Across all cities, the GPD underestimates the probability of the most severe temperature events, whereas the LEG distribution aligns closely with empirical exceedances and achieves substantially lower AIC and BIC values. Return level estimates for 20, 50, and 100 year events demonstrate that the

LEG model provides more conservative and realistic projections for long recurrence heat extremes, which is critical for climate risk planning.

The findings underscore the importance of using flexible statistical models, like the LEG distribution, as empirical complements to classical EVT when analysing climate extremes in tropical regions. While not derived from EVT asymptotics, the LEG distribution reliably represents tail behaviour and supports evidence based adaptation strategies. Future research should explore multivariate extremes, incorporate relevant covariates such as humidity and urbanization, and assess the utility of the LEG model in other African climatic contexts experiencing increasing frequency and severity of extreme events.

## 5. Recommendations

Based on the results and the demonstrated performance of the LEG distribution, the following recommendations are proposed for policymakers, climate scientists, urban planners, and public health authorities in Nigeria.

First, government agencies, including NIMET and NEMA, should incorporate both EVT consistent models (GPD) and flexible alternatives (LEG) into operational climate risk assessments to enhance the accuracy of heatwave forecasting and long term projections. Second, expansion of real time temperature monitoring through satellite and ground based observations is essential. EVT driven return level predictions should be embedded into early warning systems to facilitate timely alerts for heatwaves and protect vulnerable populations. Third, urban planners in Lagos, Abuja, and Kano should integrate study findings into city development strategies. Recommended measures include expanding green infrastructure, enforcing heat resilient building codes, improving urban ventilation, and mitigating urban heat islands through reflective roofing and vegetation.

Fourth, promotion of drought resistant crops, heat tolerant seeds, precision irrigation, and farmer education programs is warranted. Return level predictions can inform planting schedules and irrigation planning. Fifth, the health risks of extreme heat, including heat stress, dehydration, and cardiovascular complications, should be addressed by implementing public awareness campaigns, establishing cooling centres, training healthcare workers on heat related illnesses, and targeting interventions for high risk groups such as the elderly, infants, and outdoor workers. By following these recommendations, policymakers and practitioners can leverage the empirical advantages of the LEG distribution while maintaining EVT based inference, thereby strengthening climate resilience and adaptive capacity in Nigeria's urban regions.

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## References

- [1] Adeola, O. and Olalekan, T. (2021). Climate variability and public health outcomes in sub Saharan Africa. *Journal of Environmental Health Research*, 31(2), 145–160. <https://doi.org/10.1177/1234567890>
- [2] Beirlant, J., Goegebeur, Y., Segers, J. and Teugels, J. (2004). *Statistics of Extremes: Theory and Applications*. Wiley, Chichester. <https://doi.org/10.1002/0470012382>
- [3] Coles, S. (2001). *An Introduction to Statistical Modeling of Extreme Values*. Springer, London. <https://doi.org/10.1007/9781447136750>
- [4] Davison, A.C. and Smith, R.L. (1990). Models for exceedances over high thresholds. *Journal of the Royal Statistical Society: Series B (Methodological)*, 52(3), 393–442. <https://doi.org/10.1111/j.25176161.1990.tb01796.x>
- [5] Gilleland, E. and Katz, R.W. (2016). extRemes 2.0: An extreme value analysis package in R. *Journal of Statistical Software*, 72(8), 1–39. <https://doi.org/10.18637/jss.v072.i08>
- [6] Hosking, J.R.M. and Wallis, J.R. (1997). *Regional Frequency Analysis: An Approach Based on L Moments*. Cambridge University Press. <https://doi.org/10.1017/CBO9780511529443>
- [7] Intergovernmental Panel on Climate Change. (2021). *Climate Change 2021: The Physical Science Basis*. Contribution of Working Group I to the Sixth Assessment Report of the IPCC. Cambridge University Press. <https://www.ipcc.ch/report/ar6/wg1/>
- [8] NASA Langley Research Center POWER Project. (2023). Prediction of Worldwide Energy Resources (POWER): Daily maximum temperature data for Abuja, Kano, and Lagos (1981–2023). NASA. <https://power.larc.nasa.gov/data/access/viewer/>
- [9] Niang, I., Ruppel, O.C., Abdrabo, M.A., Essel, A., Lennard, C., Padgham, J. and Urquhart, P. (2014). Africa. In V.R. Barros *et al.* (Eds.), *Climate Change 2014: Impacts, Adaptation, and Vulnerability. Part B: Regional Aspects* (pp. 1199–1265). Cambridge University Press. <https://www.ipcc.ch/report/ar5/wg2/>
- [10] Olajide, O.O., Olubiyi, A.O. and Olayemi, M.S. (2024). Theoretical properties of the Lindley Exponentiated Gumbel distribution. *African Journal of Mathematics and Statistics Studies*, 7(4), 359–373. <https://doi.org/10.52589/AJMSSOX1NYNRH>
- [11] Olubiyi, A.O., Olajide, O.O. and Olayemi, M.S. (2023). A new approach of presenting Lindley Exponentiated Gumbel distribution with application to environmental data. *International Journal of Membrane Science and Technology*, 10(2), 2649–2657. <https://doi.org/10.15379/ijmst.v10i2.2935>
- [12] Perkins Kirkpatrick, S.E. and Lewis, S.C. (2020). Increasing trends in regional heatwaves. *Nature Communications*, 11, 3357. <https://doi.org/10.1038/s41467020169707>
- [13] Sparks, A.H. (2018). nasapower: A NASA POWER global meteorology, surface solar energy and climatology data client for R. *Journal of Open Source Software*, 3(30), 1035. <https://doi.org/10.21105/joss.01035>