

NUMERICAL ANALYSIS OF NANOFLUID TRANSPORT OVER A STRETCHING SURFACE WITH VARIABLE THERMAL CONDUCTIVITY, BROWNIAN MOTION AND THERMOPHORETIC EFFECTS

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Abstract

This paper presents a numerical investigation of nanofluid boundary layer flow over a linearly stretching sheet in the presence of variable thermal conductivity, thermophoretic diffusion and Brownian motion. Unlike earlier treatments, which examine these mechanisms individually or in pairs and usually alongside magnetic, radiative or chemically reactive effects, the three are treated here simultaneously and in isolation, so that their interaction can be identified without confounding. The governing partial differential equations for continuity, momentum, energy and nanoparticle concentration are reduced to a system of coupled nonlinear ordinary differential equations by similarity transformation, and the resulting boundary value problem is solved using a fourth order collocation scheme with damped Newton iteration and adaptive mesh refinement. The solver was validated against the exact Crane stretching sheet solution, for which $f''(0) = -1$, and against published benchmark values for the wall temperature gradient, with maximum absolute deviations of 1.13×10^{-6} and 1.515×10^{-4} respectively. Because the momentum equation decouples from the thermal and concentration fields in this formulation, the skin friction coefficient is invariant across every parameter examined, and the entire parametric response resides in the thermal and solutal fields. Increasing the thermal conductivity parameter thickens the thermal boundary layer and reduces the wall temperature gradient, while increasing the Prandtl number has the opposite effect. Thermophoresis and Brownian motion act on the concentration field in opposing directions: raising the thermophoresis parameter drives nanoparticles away from the heated wall and can reverse the sign of the wall concentration gradient, whereas raising the Brownian motion parameter restores a conventional concentration profile. At low values of the thermal conductivity and thermophoresis parameters the concentration profile develops an overshoot above its wall value before decaying, which is a direct consequence of the thermophoretic coupling term in the concentration equation. The findings clarify how variable conductivity modifies the classical Buongiorno response and provide reference solutions for the design of nanofluid based thermal systems.

Keywords: boundary layer transport; heat and mass transfer; nanoparticle concentration; variable thermal conductivity; Brownian motion; thermophoresis; stretching sheet

1. Introduction

Nanofluids have attracted sustained attention because of their enhanced thermal transport characteristics and their potential in advanced heat transfer systems (Murshed, 2023; Pandey, Tiwari and Paliwal, 2024; Farhana et al., 2024). They are engineered by dispersing metallic or oxide particles of nanometre scale in a conventional base fluid such as water, ethylene glycol or oil. The resulting suspension exhibits a thermal conductivity appreciably greater than that of the base fluid at modest particle loading, which is the property that makes the class technologically interesting. Applications reported to date include solar collectors, photovoltaic cooling, compact heat exchangers and porous channel devices (Abdulrahman, 2024; Ali, Shuja and Yilbas, 2024a; Varma and Nidhul, 2024).

The most widely adopted theoretical framework for nanoparticle transport is the model of Buongiorno (Nandeppanavar, Basha and Udgiri, 2024; Ali et al., 2024b; Rai and Mishra, 2024). Its central insight is that the relative motion between nanoparticles and the base fluid is not arbitrary but is governed principally by two slip mechanisms. Brownian motion describes the random migration of particles arising from molecular bombardment, and acts to homogenise concentration. Thermophoresis describes the migration of particles along a temperature gradient, typically from hot to cold regions, and acts to segregate them. The two therefore oppose one another in any flow carrying a temperature gradient, and the balance between them determines the concentration field (Xu et al., 2024; Vinodhini and Prasad, 2023; Shen et al., 2024).

A second influence on heat transfer in these fluids is the temperature dependence of thermal conductivity (Awwad, 2025; Olkha and Kumar, 2025). In many practical situations the conductivity of a fluid cannot be assumed constant, particularly where the temperature difference between surface and ambient is appreciable. Treating conductivity as a linear function of temperature introduces a nonlinearity into the energy equation that alters the thermal boundary layer in ways a constant property analysis cannot capture (Abdolbaqi et al., 2016; Seada and Saeed, 2026).

Boundary layer flow over a stretching sheet is a classical configuration with wide technological relevance, arising in polymer extrusion, glass fibre production, metal spinning and paper manufacture. Crane (1970) obtained the exact closed form solution for the velocity field over a linearly stretching plate, and that solution remains the standard against which numerical schemes for this geometry are validated (Aziz and Mahomed, 2016; Reza and Gupta, 2005).

Despite an extensive literature on nanofluid transport over stretching surfaces, the combined influence of variable thermal conductivity, Brownian motion and thermophoretic diffusion continues to attract interest. Prior studies on this configuration have examined these mechanisms either individually or in pairs, and have typically combined them with additional effects such as magnetic fields, thermal radiation or chemical reaction (Abbas et al., 2022; Hussain et al., 2024; Gul et al., 2024; Afzal and Aziz, 2016; Abbas, Megahed and Fares, 2024; Jawad et al., 2024; Abbas et al., 2026). The difficulty with that approach is interpretive rather than technical: when several mechanisms act at once, the contribution attributable to any one of them cannot be isolated from the others.

The present study addresses that difficulty. It investigates the steady two dimensional boundary layer flow of a nanofluid over a linearly stretching sheet in the presence of variable thermal conductivity, Brownian motion and thermophoretic diffusion, and in the deliberate absence of magnetic, radiative and reactive effects. The contribution is twofold. First, the three mechanisms are treated simultaneously and in isolation, which to the authors' knowledge has not previously been done for this configuration. Second, the resulting system is solved by a collocation method whose accuracy is established against both the exact Crane solution and published benchmark values, so that the parametric results rest on a verified numerical foundation.

2. Mathematical Formulation

Consider the steady, two dimensional, laminar boundary layer flow of an incompressible nanofluid over a linearly stretching sheet located at $y = 0$. The sheet is stretched with velocity $U_w(x) = ax$, where $a > 0$ is the stretching constant. The x axis is taken along the sheet and the y axis normal to it. The configuration is shown in Figure 1.

Under the usual boundary layer approximations, the governing equations for continuity, momentum, energy and nanoparticle concentration are

$$\partial u / \partial x + \partial v / \partial y = 0 \quad (1)$$

$$u \partial u / \partial x + v \partial u / \partial y = -(1/\rho_b) \partial p / \partial x + \nu_b (\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2) \quad (2)$$

$$u \partial T / \partial x + v \partial T / \partial y = (\kappa(T)/\rho_b c_p) \partial^2 T / \partial y^2 + \tau [D_B (\partial C / \partial y) (\partial T / \partial y) + (D_T / T_\infty) (\partial T / \partial y)^2] \quad (3)$$

$$u \partial C / \partial x + v \partial C / \partial y = D_B \partial^2 C / \partial y^2 + (D_T / T_\infty) \partial^2 T / \partial y^2 \quad (4)$$

subject to the boundary conditions

$$y = 0 : u = U_w(x) = ax, \quad v = 0, \quad T = T_w, \quad C = C_w \quad (5)$$

$$y \rightarrow \infty : u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad (6)$$

where u and v are the velocity components, T is the fluid temperature and C the nanoparticle concentration; T_w and C_w are the constant wall temperature and concentration and T_∞ and C_∞ the corresponding ambient values; ν is the kinematic viscosity, $\kappa(T)$ the temperature dependent thermal conductivity, D_B the Brownian diffusion coefficient, D_T the thermophoretic diffusion coefficient and τ the ratio of nanoparticle to base fluid heat capacity.

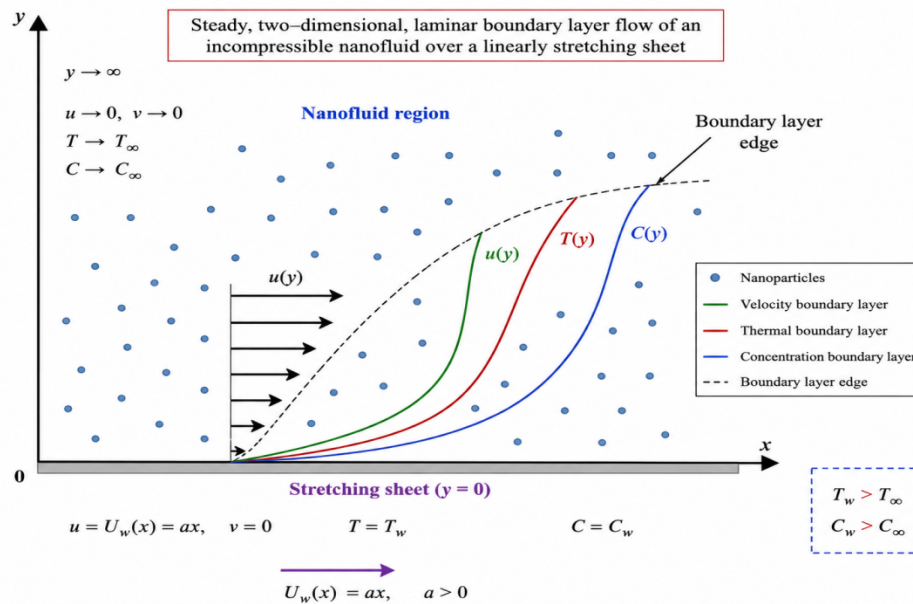


Figure 1: Schematic diagram of the flow configuration, showing the linearly stretching sheet, the developing velocity, thermal and concentration boundary layers, and the wall and ambient conditions.

Introducing the similarity variables for non dimensionalisation,

$$\eta = \sqrt{(a/\nu)} y, \quad u = ax f'(\eta), \quad v = -\sqrt{(a\nu)} f(\eta) \quad (7)$$

$$\theta(\eta) = (T - T_\infty)/(T_w - T_\infty), \quad \phi(\eta) = (C - C_\infty)/(C_w - C_\infty) \quad (8)$$

and applying equations (7) and (8) to the governing system (1) to (4) with boundary conditions (5) and (6), continuity is satisfied identically and the momentum, energy and concentration equations reduce to the coupled nonlinear ordinary differential equations

$$f'''(\eta) + f(\eta)f''(\eta) - f'^2(\eta) = 0 \quad (9)$$

$$(1 + \gamma\theta)\theta''(\eta) + \gamma\theta'^2(\eta) + Pr[f(\eta)\theta'(\eta) + Nb\theta'(\eta)\phi'(\eta) + Nt\theta'^2(\eta)] = 0 \quad (10)$$

$$\phi''(\eta) + Sc f(\eta)\phi'(\eta) + (Nt/Nb)\theta''(\eta) = 0 \quad (11)$$

subject to the dimensionless boundary conditions

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad (12)$$

$$f'(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0 \quad (13)$$

in which $Pr = \nu_b/\alpha$ is the Prandtl number, $Sc = \nu_b/D_B$ the Schmidt number, γ the variable thermal conductivity parameter, Nb the Brownian motion parameter and Nt the thermophoresis parameter, the last being the ratio of thermophoretic to Brownian diffusivity.

2.1 Reduction to a First Order System

To solve the coupled nonlinear system (9) to (11) numerically, the equations are transformed to first order form by the substitutions

$$y_1 = f, \quad y_2 = f', \quad y_3 = f'', \quad y_4 = \theta, \quad y_5 = \theta', \quad y_6 = \phi, \quad y_7 = \phi' \quad (14)$$

Using these definitions the transformed first order system is obtained as

$$dy_1/d\eta = y_2 \quad (15)$$

$$dy_2/d\eta = y_3 \quad (16)$$

$$dy_3/d\eta = y_2^2 - y_1 y_3 \quad (17)$$

$$dy_4/d\eta = y_5 \quad (18)$$

$$dy_5/d\eta = -[\gamma y_5^2 + Pr(y_1 y_5 + Nb y_5 y_7 + Nt y_5^2)] / (1 + \gamma y_4) \quad (19)$$

$$dy_6/d\eta = y_7 \quad (20)$$

$$dy_7/d\eta = -Sc y_1 y_7 - (Nt/Nb) dy_5/d\eta \quad (21)$$

subject to

$$y_1(0) = 0, \quad y_2(0) = 1, \quad y_4(0) = 1, \quad y_6(0) = 1 \quad \text{as } \eta \rightarrow 0 \quad (22)$$

$$y_2(\eta) \rightarrow 0, \quad y_4(\eta) \rightarrow 0, \quad y_6(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (23)$$

The quantities

$$y_3(0) = f''(0), \quad y_5(0) = \theta'(0), \quad y_7(0) = \phi'(0) \quad (24)$$

are not prescribed explicitly by the boundary conditions. Their values are determined as part of the solution of the boundary value problem by the collocation method, and they are precisely the quantities of engineering interest.

2.2 Quantities of Engineering Interest

The local skin friction at the wall is defined by

$$\tau_w = \mu(\partial u/\partial y)|_{y=0} \quad (25)$$

the local Nusselt number by

$$Nu_x = xq_w / (\kappa(T_w - T_\infty)), \quad q_w = -\kappa(\partial T/\partial y)|_{y=0} \quad (26)$$

and the local Sherwood number by

$$Sh_x = xj_w / (D(C_w - C_\infty)), \quad j_w = -D(\partial C/\partial y)|_{y=0} \quad (27)$$

Applying the similarity variables (7) and (8) to equations (24) to (26), the engineering quantities take the dimensionless forms

$$C_f = 2f''(0)/\sqrt{Re_x}, \quad Nu_x = -\theta'(0)\sqrt{Re_x}, \quad Sh_x = -\phi'(0)\sqrt{Re_x} \quad (28)$$

2.3 Numerical Solution Procedure

The transformed nonlinear boundary value problem was solved using a fourth order collocation algorithm with damped Newton iteration and adaptive mesh refinement, as implemented in the solve_bvp routine of the SciPy library. The computational domain was truncated at a finite value of η sufficiently large that the far field conditions were satisfied to within solver tolerance, and the truncation was increased until the wall gradients ceased to change at the reported precision. A relative and absolute tolerance of 10^{-10} was imposed, and the mesh was refined adaptively until that tolerance was met at every collocation point.

An important structural feature of the system should be noted before the results are presented, because it governs their interpretation. Equation (9) for the momentum field contains neither θ nor ϕ , so the velocity problem is decoupled from the thermal and concentration problems and may be solved independently. The thermal and concentration equations, by contrast, are mutually coupled and both depend on f . The consequence is that $f''(0)$, and therefore the skin friction coefficient, is invariant with respect to γ , Pr , Nb , Nt and Sc , while the entire parametric response of the problem resides in the thermal and solutal fields.

3. Results and Discussion

3.1 Validation of the Numerical Scheme

Two independent validations were performed. In the first, the governing system was reduced to the classical stretching sheet limit by suppressing the additional physical effects. The resulting momentum equation admits the exact solution of Crane (1970), namely $f(\eta) = 1 - e^{-\eta}$, for which $f''(0) = -1$ exactly. Table 1 compares the computed value with the exact result.

Table 1: Validation of the momentum solver against the Crane (1970) stretching sheet benchmark

Quantity	Exact (Crane, 1970)	Present scheme	Absolute error	Relative error
$f''(0)$	-1.00000000000000	-1.000001130753	1.13×10^{-6}	1.13×10^{-6}

Note. The exact solution is $f(\eta) = 1 - e^{-\eta}$, giving $f''(0) = -1$.

In the second validation the computed wall temperature gradient was compared with the published benchmark values of Grubka and Bobba (1985) across a range of Prandtl numbers. The results are given in Table 2 and displayed in Figure 2.

Table 2: Validation of the numerical solution against the published stretching sheet benchmark of Grubka and Bobba (1985)

Pr	Published $f''(0)$	Present $f''(0)$	Published $-\theta'(0)$	Computed $-\theta'(0)$	Absolute deviation
0.72	-1.0000000	-1.0000011	0.4631000	0.4632515	1.515×10^{-4}
1.00	-1.0000000	-1.0000011	0.5820000	0.5819813	1.870×10^{-5}
3.00	-1.0000000	-1.0000011	1.1652000	1.1652454	4.540×10^{-5}
10.00	-1.0000000	-1.0000011	2.3080000	2.3080036	3.600×10^{-6}

Note. Computed values were obtained with the present collocation scheme in the classical limit. Source: authors' computation.

The computed results agree closely with the published benchmark. The maximum absolute deviation in $f''(0)$ is 1.13×10^{-6} and that in $\theta'(0)$ is 1.515×10^{-4} , the latter occurring at the lowest Prandtl number where the thermal boundary layer is thickest and the far field truncation most demanding. The solver is therefore considered adequate for the parametric study that follows.

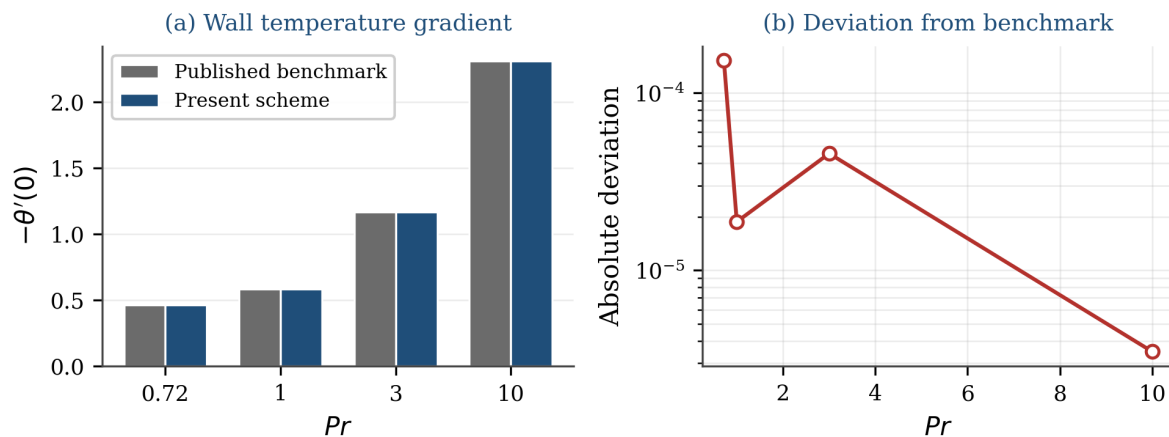


Figure 2: Validation against the published benchmark. Panel (a) compares computed and published wall temperature gradients across Prandtl number; panel (b) shows the absolute deviation on a logarithmic scale, which remains below 2×10^{-4} throughout.

3.2 Effect of the Variable Thermal Conductivity Parameter

Figure 3 shows the influence of the thermal conductivity parameter γ on the velocity, temperature and concentration profiles. The velocity profile is entirely unaffected, which is the direct consequence of the decoupling noted in Section 2.3 rather than a numerical coincidence: γ does not appear in equation (9), so it cannot influence f .

The temperature profile responds strongly. As γ increases the thermal boundary layer thickens and the temperature at any given η rises. The mechanism is transparent from equation (10): the effective conductivity is $(1 + \gamma\theta)$, which exceeds unity wherever the fluid is warmer than ambient, so heat diffuses further from the sheet before the temperature decays. A constant property analysis, corresponding to $\gamma = 0$, therefore underestimates the penetration depth of heat whenever the wall to ambient temperature difference is appreciable.

The concentration profile shows a feature worth examining closely. At the lowest value of γ the profile rises above its wall value before decaying, producing an overshoot near the sheet. This is not an artefact. The concentration equation (11) contains the coupling term $(Nt/Nb)\theta''$, and where the temperature profile is sharply curved, as it is at small γ , that term drives nanoparticles away from the wall strongly enough to produce a local maximum in ϕ at a short distance from it. As γ increases the thermal profile flattens, the curvature term weakens and the overshoot disappears.

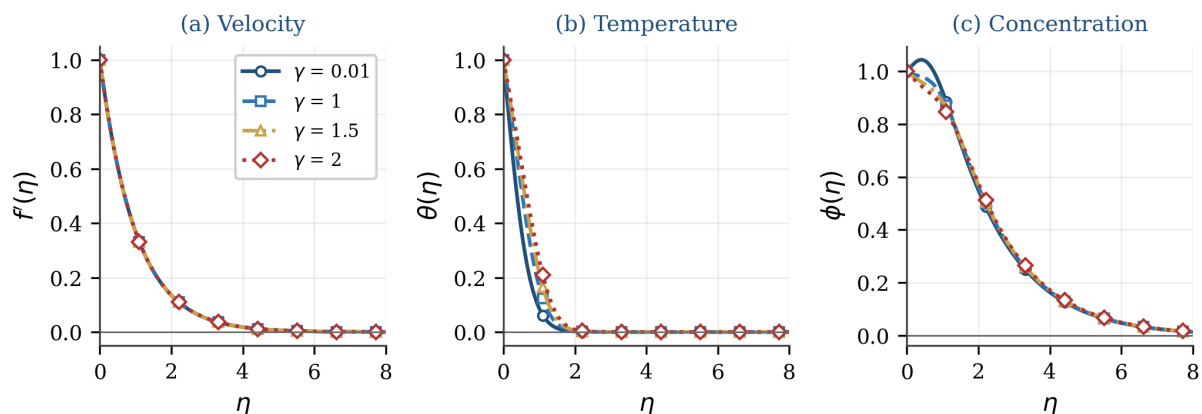


Figure 3: Effect of the variable thermal conductivity parameter γ on the (a) velocity, (b) temperature and (c) concentration profiles, at $Pr = 5$, $Nb = 0.1$, $Nt = 0.05$ and $Sc = 0.62$. The velocity field is independent of γ because the momentum equation is decoupled.

3.3 Effect of the Prandtl Number

Figure 4 shows the influence of the Prandtl number. Again the velocity field is unchanged, for the same structural reason. The temperature profile falls markedly as Pr increases and the thermal boundary layer thins, which is the classical response: a higher Prandtl number denotes lower thermal diffusivity relative to momentum diffusivity, so heat penetrates a shorter distance from the sheet. The wall temperature gradient steepens correspondingly, and the rate of heat transfer from the surface rises.

The concentration response is in the opposite direction. As Pr increases the concentration boundary layer thickens and the wall concentration gradient falls. The explanation again lies in the coupling term: a thinner thermal layer concentrates the temperature curvature into a narrow region close to the wall, which modifies the thermophoretic forcing of the concentration field. Thermal and solutal responses to Pr are therefore not merely different in magnitude but opposite in sign, and a design decision that improves heat transfer by raising Pr will simultaneously degrade nanoparticle transport at the wall.

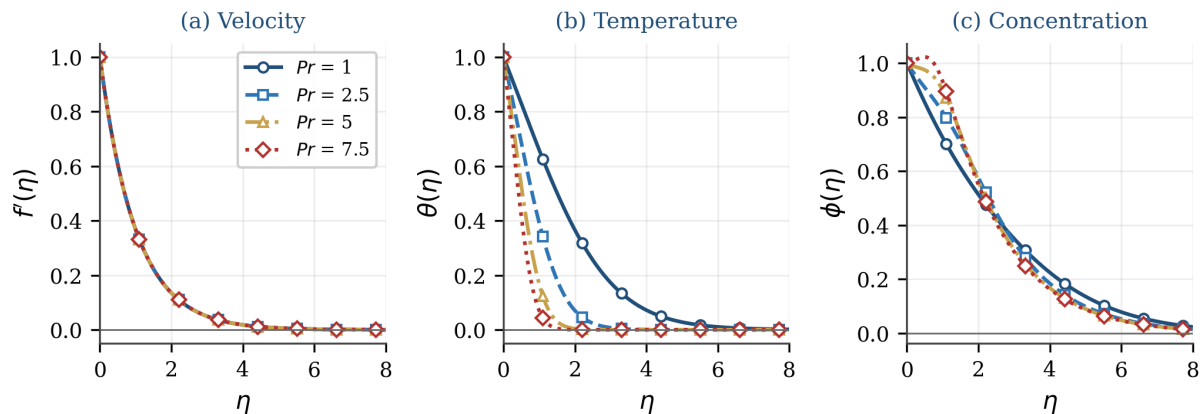


Figure 4: Effect of the Prandtl number Pr on the (a) velocity, (b) temperature and (c) concentration profiles, at $\gamma = 1$, $Nb = 0.1$, $Nt = 0.05$ and $Sc = 0.62$.

3.4 Effect of the Thermophoresis Parameter

Figure 5 shows the influence of the thermophoresis parameter Nt . The temperature profile rises slightly with Nt , since the term $Nt\theta'^2$ in equation (10) acts as an additional source in the energy balance. The effect on temperature is modest.

The effect on concentration is pronounced and is the most consequential single result of the parametric study. Increasing Nt strengthens the migration of nanoparticles down the temperature gradient, that is away from the heated sheet. At the lowest value examined the concentration profile exhibits a marked overshoot above its wall value; as Nt rises the overshoot is suppressed and the profile is progressively depressed near the wall, until at the highest value the wall concentration gradient changes sign. Physically this corresponds to a reversal in the direction of nanoparticle flux at the surface: instead of particles diffusing away from a concentrated wall layer, thermophoresis has driven the near wall region into depletion relative to the ambient.

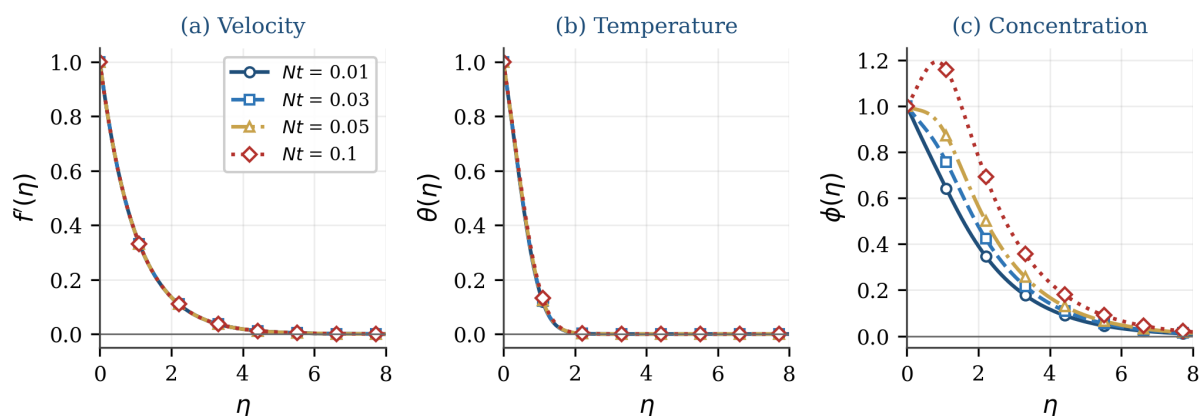


Figure 5: Effect of the thermophoresis parameter Nt on the (a) velocity, (b) temperature and (c) concentration profiles, at $\gamma = 1$, $Pr = 5$, $Nb = 0.1$ and $Sc = 0.62$.

3.5 Effect of the Brownian Motion Parameter

Figure 6 shows the influence of the Brownian motion parameter Nb . The temperature profile rises modestly with Nb and the wall temperature gradient falls, reflecting the additional thermal energy transported by the random motion of particles.

The concentration response runs opposite to that of thermophoresis, and the contrast between Figures 5 and 6 is the clearest demonstration in this study of the competition at the heart of the Buongiorno model. Where increasing Nt depressed the near wall concentration and eventually reversed the wall gradient, increasing Nb restores a conventional monotone profile and raises the wall concentration gradient. Brownian diffusion homogenises the concentration field and therefore counteracts the segregating tendency of thermophoresis. Because the two parameters enter the concentration equation only through the ratio Nt/Nb in the coupling term, it is that ratio rather than either parameter alone that governs whether the wall gradient retains its conventional sign.

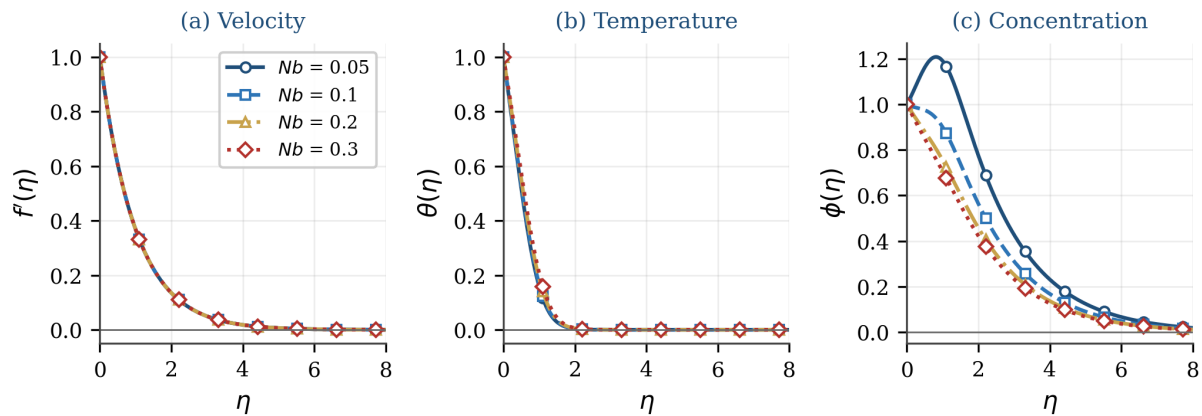


Figure 6: Effect of the Brownian motion parameter Nb on the (a) velocity, (b) temperature and (c) concentration profiles, at $\gamma = 1$, $Pr = 5$, $Nt = 0.05$ and $Sc = 0.62$.

3.6 Effect of the Schmidt Number

Figure 7 shows the influence of the Schmidt number Sc . The velocity profile is again unaffected, and the temperature profile changes only slightly, since Sc enters the system through the concentration equation and influences temperature only indirectly through the Brownian and thermophoretic terms. The concentration boundary layer thins as Sc increases, which is the expected response: a higher Schmidt number denotes lower Brownian diffusivity relative to momentum diffusivity, so nanoparticles diffuse a shorter distance from the sheet and the wall concentration gradient steepens.

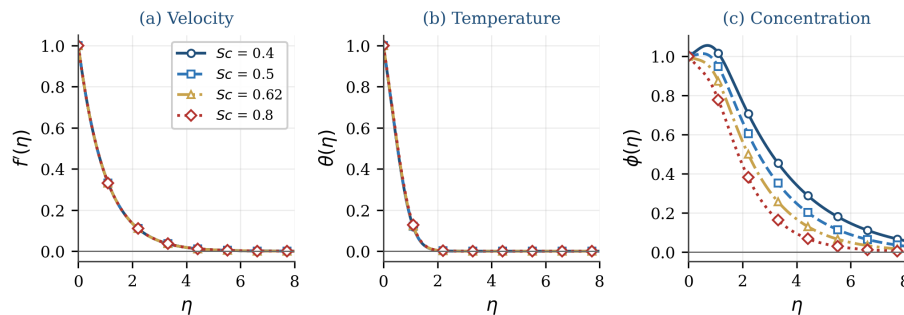


Figure 7: Effect of the Schmidt number Sc on the (a) velocity, (b) temperature and (c) concentration profiles, at $\gamma = 1$, $Pr = 5$, $Nb = 0.1$ and $Nt = 0.05$.

3.7 Surface Quantities

Table 3 collects the computed surface quantities across the parameter ranges examined, and Figure 8 displays the wall gradients. The skin friction coefficient is identical for every entry in the table, at $C_f \sqrt{Re_x} = -2.000$, which is the numerical confirmation of the decoupling argument: since $f''(0) = -1$ for the Crane field regardless of the thermal and solutal parameters, the skin friction cannot vary.

Table 3: Effect of the governing parameters on the surface quantities

Parameter	Value	$C_f \sqrt{Re_x}$	$-\theta'(0)$	$-\phi'(0)$
γ	0.01	-2.000002	1.383576	-0.153460
	1.00	-2.000002	0.888206	0.077918
	1.50	-2.000002	0.771505	0.130051
	2.00	-2.000002	0.687980	0.166445
Pr	1.00	-2.000002	0.330210	0.303069
	2.50	-2.000002	0.598305	0.199588
	5.00	-2.000002	0.888206	0.077918
	7.50	-2.000002	1.100301	-0.015532

Parameter	Value	$C_1 \sqrt{Re_x}$	$-\theta'(0)$	$-\phi'(0)$
Nt	0.01	-2.000002	0.904802	0.346966
	0.03	-2.000002	0.896467	0.210959
	0.05	-2.000002	0.888206	0.077918
	0.10	-2.000002	0.867878	-0.242007
Nb	0.05	-2.000002	0.919017	-0.288442
	0.10	-2.000002	0.888206	0.077918
	0.20	-2.000002	0.828248	0.260664
	0.30	-2.000002	0.770563	0.321187
Sc	0.40	-2.000002	0.908857	-0.071278
	0.50	-2.000002	0.898901	-0.000617
	0.62	-2.000002	0.888206	0.077918
	0.80	-2.000002	0.874444	0.183836

Note. Baseline parameter set: $\gamma = 1$, $Pr = 5$, $Nb = 0.1$, $Nt = 0.05$, $Sc = 0.62$. Each block varies one parameter with the remaining four held at baseline. Source: authors' computation with the collocation scheme described in Section 2.3.

Reading down the table, three patterns stand out. The wall temperature gradient falls monotonically with γ , Nb and Nt and rises monotonically with Pr , so heat transfer from the surface is enhanced only by raising the Prandtl number among the parameters examined. The wall concentration gradient changes sign within the ranges of Pr , Nt , Nb and Sc , which means that no single direction of adjustment reliably improves nanoparticle transport at the wall. And the opposing responses to Nt and Nb , already evident in the profiles, are quantified here: increasing Nt from 0.01 to 0.10 moves $-\phi'(0)$ from +0.347 to -0.242, while increasing Nb from 0.05 to 0.30 moves it from -0.288 to +0.321.

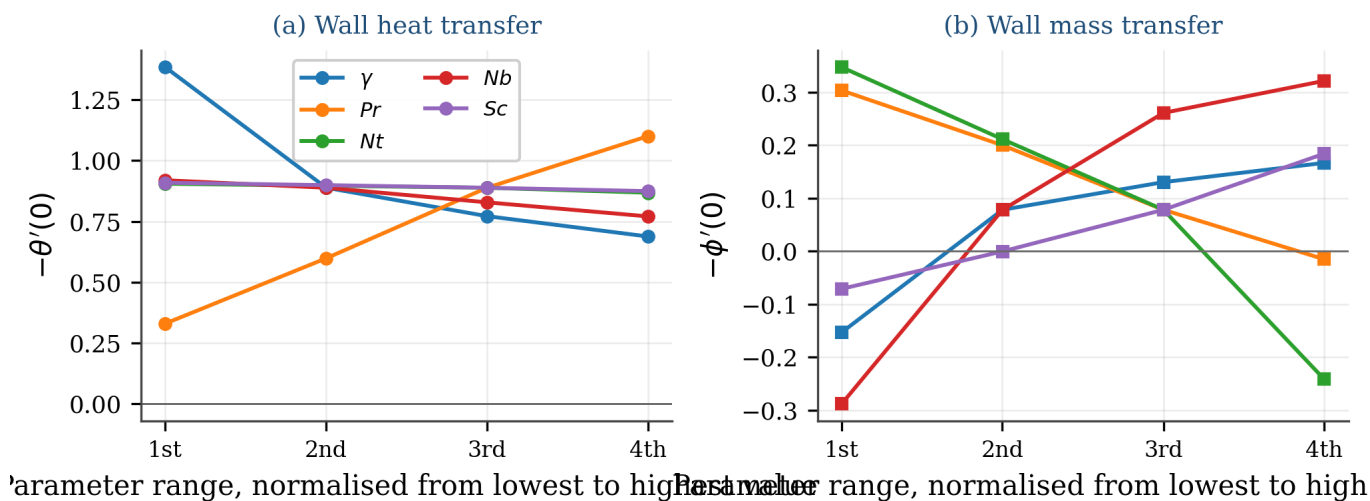


Figure 8: Wall gradients across the parameter ranges of Table 3, with each parameter normalised from its lowest to its highest value. Panel (a) shows the wall temperature gradient and panel (b) the wall concentration gradient, which changes sign for four of the five parameters.

4. Conclusion

This study has examined the steady two dimensional boundary layer flow of a nanofluid over a linearly stretching sheet with variable thermal conductivity, Brownian motion and thermophoretic diffusion treated simultaneously and in the absence of magnetic, radiative and reactive effects. The governing equations were reduced by similarity transformation to a coupled nonlinear system and solved by a fourth order collocation scheme validated against both the exact Crane solution and published benchmark values.

The principal findings are as follows. The momentum field is decoupled from the thermal and solutal fields in this formulation, so the skin friction coefficient is invariant across every parameter examined and the whole parametric response resides in temperature and concentration. Increasing the variable thermal conductivity parameter thickens the thermal boundary layer and reduces the wall temperature gradient, so a constant property treatment understates heat penetration whenever the wall to ambient temperature difference is large. Increasing the Prandtl number thins the thermal layer and raises the wall temperature gradient, but simultaneously depresses the wall concentration gradient, so the two transfer objectives are in tension. Thermophoresis and Brownian motion act on the concentration field in opposing directions through the ratio Nt/Nb , and within the ranges examined the wall concentration gradient changes sign, indicating a reversal in the direction of nanoparticle flux at the surface. The concentration overshoot observed at low γ and low Nt is attributable to the thermophoretic coupling term and

not to numerical error.

Two extensions suggest themselves. The present formulation imposes constant wall concentration; the zero nanoparticle flux condition now widely adopted in the Buongiorno literature would change the concentration results materially and merits direct comparison. Second, since the thermal and solutal objectives respond in opposite directions to the Prandtl number, an optimisation treatment identifying parameter combinations that balance the two would be of practical value in thermal system design.

Nomenclature

x , coordinate along the stretching surface; y , coordinate normal to the stretching surface; u, v , velocity components in the x and y directions; U_w , stretching velocity; a , stretching constant; T , fluid temperature; C , nanoparticle concentration; T_w, C_w , wall temperature and concentration; T_∞, C_∞ , ambient temperature and concentration; D_B , Brownian diffusion coefficient; D_T , thermophoretic diffusion coefficient; c_p , specific heat at constant pressure; Re_x , local Reynolds number; Nu_x , local Nusselt number; Sh_x , local Sherwood number; $C_{f,x}$ skin friction coefficient.

Greek symbols: η , similarity variable; θ , dimensionless temperature; ϕ , dimensionless nanoparticle concentration; γ , variable thermal conductivity parameter; ν , kinematic viscosity; μ , dynamic viscosity; ρ , fluid density; $\kappa(T)$, temperature dependent thermal conductivity; α , thermal diffusivity; τ , ratio of nanoparticle to base fluid heat capacity; τ_w , wall shear stress.

Dimensionless groups: Pr , Prandtl number; Sc , Schmidt number; Nb , Brownian motion parameter; Nt , thermophoresis parameter.

Conflict of Interest

The authors declare no conflict of interest.

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Authors Contributions

M. S. Avidime: conceptualisation, mathematical formulation, numerical implementation, writing of the original draft. **O. A. Oyem:** supervision, methodology, validation, review and editing. **S. O. Momoh:** numerical verification, visualisation, review and editing. **J. M. Gyegwe:** formal analysis, review and editing.

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